



Duality After Supersymmetry Breaking

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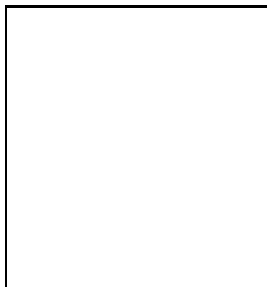
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DUALITY AFTER SUPERSYMMETRY BREAKING

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Starting with two supersymmetric dual theories, we imagine adding a chiral perturbation that breaks supersymmetry dynamically. At low energy we then get two theories with soft supersymmetry-breaking terms that are generated dynamically. With a canonical Kähler potential, some of the scalars of the “magnetic” theory typically have negative mass-squared, and the vector-like symmetry is broken. Since for large supersymmetry breaking the “electric” theory becomes ordinary QCD, the two theories are then incompatible. For small supersymmetry breaking, if duality still holds, the magnetic theory analysis implies specific patterns of chiral symmetry breaking in supersymmetric QCD with small soft masses.

1 Introduction

The low energy behavior of gauge theories, which are often strongly-coupled in the infrared, becomes more tractable in the presence of supersymmetry. While supersymmetry doubles the spectrum of the theory, and in particular, requires the existence of scalars to accompany the chiral “matter” fermions of the theory, it also results in various constraints on the system. This has allowed for the discovery of interesting patterns of low-energy dynamics in $\mathcal{N} = 1$ gauge theories. In particular, we know of dual pairs of theories¹: theories with different gauge symmetry and matter content, that have the same physics at low energies. Clearly, the two theories must have the same global symmetries, and satisfy ‘t Hooft anomaly matching conditions: the massless fermions give the same contributions to the various global triangle anomalies, in the two theories. Furthermore, in some cases, while one theory is strongly coupled in the infrared, its dual is weakly coupled. In analogy with electric-magnetic duality, the two dual theories are usually referred to as “electric” and “magnetic”.

Alas, the real world is not supersymmetric at low energies, and so it is tempting to ask whether this exciting phenomenon survives in non-supersymmetric theories^{2–6}. Starting with two (supersymmetric) dual theories, we wish to introduce supersymmetry breaking into the two

theories, and to study the resulting infrared dynamics. The first question one encounters is how to introduce supersymmetry breaking into the system. One way to do this is to break supersymmetry explicitly. For example, as in the MSSM, we could add soft supersymmetry-breaking terms, such as squark and gaugino masses, in the electric theory. However, it is not clear what these map into in the magnetic theory, since the correspondence between the two theories is only known to hold in the supersymmetric limit. In fact, even in this limit we generally only know how chiral operators map between the two theories⁷.

Instead, we will add a chiral, supersymmetric perturbation to the theory that triggers spontaneous supersymmetry breaking below a certain scale. Since supersymmetry is only spontaneously broken, and since the perturbation we add is a chiral superpotential term, we know what this perturbation maps into in the dual theory, in the limit of unbroken supersymmetry. Adding the perturbation in the dual theory, supersymmetry is spontaneously broken in this theory too. We will then have, at low energy, two theories with soft supersymmetry breaking terms that are generated dynamically. To continue the MSSM analogy, each one of the theories will now resemble a model with gauge mediated supersymmetry breaking^{8–9} (GMSB), in which the MSSM is coupled to a sector that breaks supersymmetry spontaneously, and the soft supersymmetry-breaking terms of the MSSM are generated dynamically as a result of this coupling.

We will construct the electric theory so that the scalar masses squared, m_{scalar}^2 , generated in this theory are positive. Surprisingly, however, the scalar masses squared generated in the magnetic theory, \bar{m}_{scalar}^2 , will often turn out negative. The global symmetries of the magnetic theory are then partially broken.

We will consider separately two limits. In the limit of small supersymmetry breaking, we may hope that the correspondence between the two theories persists. Thus, by studying the pattern of chiral symmetry breaking in the magnetic theory, when it is weakly coupled in the infrared, we may learn something about the chiral symmetry of the electric theory, when this theory is strongly coupled.

In the limit of large supersymmetry breaking, the electric theory we study looks like QCD with N colors and N_f flavors. The pattern of chiral symmetry breaking we find in the dual theory is incompatible with that expected for QCD. We can then conclude that the duality between the two theories no longer holds.

One key ingredient in our analysis is the Kähler potential of the dual theory, which is unknown even in the supersymmetric case. We will therefore assume, throughout our analysis, a minimal Kähler potential.

2 The model

Let us first review some elements of $\mathcal{N} = 1$ duality¹. The first example of this duality, given by Seiberg, involves an electric theory with gauge group $SU(N)$ and N_f “flavors”, that is, fields Q^i, \bar{Q}_i , in the fundamental and anti-fundamental representation respectively, with $i = 1 \dots N_f$. For $N_f \geq N + 2$, the theory has a dual, magnetic theory, with gauge group $SU(N_f - N)$, N_f flavors of dual quarks q_i, \bar{q}^i and gauge singlets M_j^i , and the superpotential

$$W = M_j^i q_i \cdot \bar{q}^j . \quad (1)$$

In the infrared, the two theories describe the same physics, with the mesons $Q^i \cdot \bar{Q}_j$ of the electric theory mapped into the fields M_j^i of the magnetic theory. The two theories have the same global symmetry, $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$, and identical global anomalies (’t Hooft anomaly conditions are matched).

It is interesting to see what happens to this picture as we change the number of flavors¹. Suppose we start with $SU(N)$ with $N_f + 1$ flavors and make the “last” flavor massive, that is,

we add a mass term $mQ^{N_f+1} \cdot \bar{Q}_{N_f+1}$ to the superpotential of the electric theory. The dual of this theory has gauge group $SU(N_f+1-N)$, N_f+1 flavors, and a superpotential that contains, among other terms,

$$M_{N_f+1}^{N_f+1} q_{N_f+1} \cdot \bar{q}^{N_f+1} + m M_{N_f+1}^{N_f+1} , \quad (2)$$

where the first term comes from eqn. (1), and the second corresponds to the mass term we added in the electric theory. As a result, the fields q_{N_f+1} , \bar{q}^{N_f+1} develop vevs, and the gauge group is broken, or “Higgsed”, to $SU(N_f-N)$. Since the “last” flavor (q_{N_f+1} , \bar{q}^{N_f+1}) is eaten, the magnetic theory becomes $SU(N_f-N)$ with N_f flavors, precisely the dual we expect for the electric theory, which below the scale m is $SU(N)$ with N_f flavors.

We now wish to introduce supersymmetry breaking into the system. We take the electric theory described above, with N colors and N_f+1 flavors, and couple it to a sector that breaks supersymmetry dynamically (the DSB sector), by introducing the superpotential coupling,

$$S Q^{N_f+1} \cdot \bar{Q}_{N_f+1} , \quad (3)$$

where S is a field of the DSB sector⁹.

In the supersymmetric limit, which is typically attained by setting some superpotential term to zero in the DSB sector, the theory has a dual description with gauge group $SU(N_f-N+1)$ and N_f+1 flavors. If the field S develops a nonzero vev, the term (3) looks like a mass term for the flavor N_f+1 , and the magnetic theory is Higgsed by one unit. At low energy then, the electric theory is $SU(N)$ and the magnetic theory is $SU(N_f-N)$, both with N_f light flavors. When supersymmetry is broken, we will assume that the auxiliary component of S , F_S , also obtains a vev. Then, in the electric theory, the masses of the heavy multiplet Q^{N_f+1} , \bar{Q}_{N_f+1} are split: while the fermions have mass S , the scalars have $m^2 = S^2 \pm F_S$. Here S and F_S stand for the appropriate vevs. As a result, supersymmetry-breaking masses are generated for the remaining fields of the electric theory. The gauginos and squarks with $i = 1 \dots N_f$ obtain masses through loops involving the heavy flavor fields. The heavy fields of the last flavor N_f+1 therefore act as “messengers” of supersymmetry breaking. In the magnetic theory, the fields q^{N_f+1} , \bar{q}_{N_f+1} , are eaten and join the heavy gauge multiplet. But again, since supersymmetry is broken, the masses of the fermion, scalar, and vector fields making up that multiplet are split by amounts proportional to F_S . The heavy gauge multiplet then acts as a messenger of supersymmetry breaking in the magnetic theory. The gauginos, squarks, and scalar M_j^i ’s with $i = 1 \dots N_f$ develop masses through loop diagrams, with the heavy messengers running in the loops.

At low energy we thus obtain two theories, with gauge groups $SU(N)$ and $SU(N_f-N)$, each with N_f flavors, that are related by Seiberg’s duality in the supersymmetric limit. When supersymmetry is broken, soft masses are generated in each theory. These masses arise through “matter messengers” in the electric theory, and through “gauge messengers” in the magnetic theory^a. Assuming a minimal Kähler potential in the dual theory, we can calculate these soft masses. We will mainly be interested in the signs of the scalar masses squared. More precisely, we can construct the theory so that m_{scalar}^2 are positive in the electric theory, and so we will focus on the signs of \bar{m}_{scalar}^2 in the magnetic theory. We will separate the discussion into two parts depending on the size of the soft masses.

3 Small supersymmetry breaking

Consider first the case of very small supersymmetry breaking, such that the soft masses are very small compared to all relevant scales in the theory. We can reliably study the magnetic theory at

^aThere are actually matter messengers in the magnetic theory as well¹⁰.

low energies when $N_f < 3N/2$, where the theory is infrared-free, or in the large N limit, where it has an infrared perturbative fixed point for N_f just above $3N/2$ ¹¹. If \bar{m}_{scalar}^2 is sufficiently small, there will be enough running from the scale at which the soft masses are generated to the scale \bar{m}_{scalar} , for the masses to reach their asymptotic behavior. We then find the following sum rule:

$$m_q^2 + m_{\bar{q}}^2 + m_M^2 \rightarrow 0, \quad (4)$$

in the deep infrared. Either the squarks or the scalar mesons therefore develop negative masses-squared!

Whether $m_q^2 < 0$ or $m_M^2 < 0$ depends on N , N_f , and the gauge and Yukawa couplings. For $m_q^2 < 0$ and $m_M^2 > 0$, (this is the case for large N , with $N_f \ll 3N/2$), the theory has a stable minimum with the global symmetry broken to $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$, or with L and R exchanged. For $m_q^2 > 0$ and $m_M^2 < 0$, the tree-level potential is unbounded from below along directions with non-zero M_j^i vevs. However, along directions with $\text{rank}(\langle M_j^i \rangle) > N$ non-perturbative effects give a non-zero potential, so that the theory will most likely slide away from the origin with the global symmetry broken to $SU(N)_V \times SU(N_f - N)_L \times SU(N_f - N)_R \times U(1)'$.

The electric theory here is supersymmetric QCD, with N colors and N_f flavors, and with very small supersymmetry-breaking soft terms. In particular, the squarks and gauginos have masses much smaller than the $SU(N)$ scale, and are not decoupled. In the range of N_f discussed above, $N_f \leq 3N/2$, this theory is strongly coupled in the infrared, and we cannot analyze it directly. However, for very small supersymmetry breaking, duality may still hold, and we may use the magnetic theory to learn something about the electric theory, at least to leading order in the supersymmetry breaking. We may then conjecture that the chiral symmetry of the electric theory is partially broken, with the maximal unbroken symmetry being either $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$, or $SU(N)_V \times SU(N_f - N)_L \times SU(N_f - N)_R \times U(1)'$. In either case, the vector-like symmetry of the electric theory is partially broken. This is possible since the theory contains light scalars.

4 Large supersymmetry breaking

We now turn to the other limiting case, that of large supersymmetry breaking. Here, the soft masses generated in each theory are large compared with the scale of the theory (assuming it is asymptotically free). The gauginos and squarks of the electric theory decouple, and this theory approaches QCD, with N colors and N_f flavors, for which we expect vector-like symmetries to remain unbroken¹².

In the magnetic theory, we have to consider both one-loop and two-loop contributions to the soft masses, since the one-loop contributions vanish at leading order in the supersymmetry-breaking parameter, while the two-loop contributions do not. There is then some region of supersymmetry breaking, where the two-loop contributions dominate. The signs of m_q^2 and m_M^2 again depend on N , N_f and the gauge and Yukawa couplings, but in the large N limit we have $m_q^2 < 0$ and $m_M^2 > 0$. Then, as discussed in the previous section, the theory has a minimum with the global symmetry broken to $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$.

For larger supersymmetry breaking, the one-loop contributions become dominant. Then we always have $m_M^2 < 0$. We immediately see that there is no region where the full chiral symmetry remains unbroken. Thus, for large N_f , such that the electric theory is infrared free, the two theories are clearly different in the infrared. Furthermore, the magnetic theory can not correspond to an infrared fixed point with the full chiral symmetry unbroken.

The sign of m_q^2 can be either positive or negative. For large $N_f - N$, it is almost always positive, and for $N_f - N = 3$ it is almost always negative. As a result, the only possible configuration that preserves the vector-like symmetry of the theory has vanishing squark vevs,

and an M_j^i vev proportional to the identity matrix. Along this direction, the tree-level potential is unbounded from below. Since supersymmetry is badly broken, we have no control over non-perturbative effects here. Still, if we estimate the nonperturbative potential by $\sim \Lambda_L^4(M)$, where $\Lambda_L(M)$ is the strong coupling scale after integrating out the quark fields which obtain masses from the meson vevs, then the potential is lifted at large scales along the direction $M \propto I$ for a certain range of N_f , and the vacuum will slide away, with different M_j^i vevs. Some of the vector symmetries of the theory will then be broken, in contradiction with what we expect for the electric theory.

5 Conclusions

We have studied the infrared behavior of theories related by Seiberg duality in the presence of supersymmetry breaking. The difficulty of not knowing what the soft supersymmetry breaking terms in one theory map into in its dual is overcome by generating the soft breaking terms in both theories by coupling them to the same sector which breaks supersymmetry spontaneously. Generating soft breaking masses in the electric theory by heavy *matter* messengers corresponds to generating soft breaking masses in the magnetic theory by heavy *gauge* messengers. Assuming a canonical Kähler potential, we found that the soft breaking scalar masses squared generated in the magnetic theory are often negative, leading to symmetry breaking in the magnetic theory.

If duality still holds approximately for small supersymmetry-breaking masses (much smaller than the strong coupling scale) the (weakly coupled) magnetic theory may be used for studying strongly coupled supersymmetric QCD with small supersymmetry-breaking masses. Our results for the magnetic theory can be roughly summarized as follows: We obtain an interesting sum rule, $m_q^2 + m_{\tilde{q}}^2 + m_M^2 = 0$ in the deep infrared, so that the masses-squared of either the dual squarks or the mesons are negative. In the region $N_f \ll 3N/2$, we find $m_q^2 < 0$, $m_M^2 > 0$ in the deep infrared. The theory has a stable minimum with the symmetry broken to $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$, or with L and R exchanged. When $m_q^2 > 0$, $m_M^2 < 0$ we find that the symmetry is broken to $SU(N)_V \times SU(N_f - N)_L \times SU(N_f - N)_R \times U(1)'$.

We also consider the large supersymmetry-breaking limit. Below the soft supersymmetry-breaking mass scale the squarks and gaugino in the electric theory decouple. The theory becomes ordinary, non-supersymmetric QCD. In the magnetic theory we typically find that either the mesons or the squarks or both obtain negative masses squared. As a result, the magnetic theory has no stable minimum with unbroken vector-like symmetries within the minimal framework we assumed. This is in contradiction to what we expect for non-supersymmetric QCD. The candidate duals we considered therefore do not describe the same low-energy physics as ordinary QCD.

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